A THEORY OF THE URBAN LAND MARKET

By William Alonso

The author is Assistant Professor of City Planning in Harvard University.

THE EARLY THEORY of rent and location concerned itself primarily with agricultural land. This was quite natural, for Ricardo and Malthus lived in an agricultural society. The foundations of the formal spatial analysis of agricultural rent and location are found in the work of J. von Thunen, who said, without going into detail, that the urban land market operated under the same principles.[1] As cities grew in importance, relatively little attention was paid to the theory of urban rents. Even the great Marshall provided interesting but only random insights, and no explicit theory of the urban land market and urban locations was developed.

Since the beginning of the twentieth century there has been considerable interest in the urban land market in America. R.M. Hurd[2] in 1903 and R. Haig[3] in the twenties tried to create a theory of urban land by following von Thunen. However, their approach copied the form rather than the logic of agricultural theory, and the resulting theory can be shown to be insufficient on its own premises. In particular, the theory failed to consider residences, which constitute the preponderant land use in urban areas.

Yet there are interesting problems that a theory of urban land must consider. There is, for instance, a paradox in American cities: the poor live near the center, on expensive land, and the rich on the periphery, on cheap land. On the logical side, there are also aspects of great interest, but which increase the difficulty of the analysis. When a purchaser acquires land, he acquires two goods (land and location) in only one transaction, and only one payment is made for the combination. He could buy the same quantity of land at another location, or he could buy more, or less land at the same location. In the

[1] *Johan von Thunen,* DER ISOLIERTE STAAT IN BEZIEHUNG AUF LANDWIRTSCHAFT UND NATIONALEKONOMIE, *1st. vol., 1826, 3d. vol. and new edition, 1863.*

[2] *Richard M. Hurd,* PRINCIPLES OF CITY LAND VALUES, *N.Y.: The Record and Guide, 1903.*

[3] *Robert M. Haig. "Toward an Understanding of the Metropolis",* QUARTERLY JOURNAL OF ECONOMICS, *XL: 3, May 1926; and* REGIONAL SURVEY OF NEW YORK AND ITS ENVIRONS, *N.Y.: New York City Plan Commission, 1927.*

analysis, one encounters, as well, a negative good (distance) with positive costs (commuting costs); or, conversely, a positive good (accessibility) with negative costs (savings in commuting). In comparison with agriculture, the urban case presents another difficulty. In agriculture, the location is extensive: many square miles may be devoted to one crop. In the urban case the site tends to be much smaller, and the location may be regarded as a dimensionless point rather than an area. Yet the thousands or millions of dimensionless points which constitute the city, when taken together, cover extensive areas. How can these dimensionless points be aggregated into two-dimensional space?

Here I will present a non-mathematical over-view, without trying to give it full precision, of the long and rather complex mathematical analysis which constitures a formal theory of the urban land market.[4] It is a static model in which change is introduced by comparative statics. And it is an economic model: it speaks of economic men, and it goes without saying that real men and social groups have needs, emotions, and desires which are not considered here. This analysis uses concepts which fit with agricultural rent theory in such a way that urban and rural land uses may be considered at the same time, in terms of a single theory. Therefore, we must examine first a very simplified model of the agricultural land market.

AGRICULTURAL MODEL

In this model, the farmers are grouped around a single market, where they sell their products. If the product is wheat, and the produce of one acre of wheat sells for $100 at the market while the costs of production are $50 per acre, a farmer growing wheat at the market would make a profit of $50 per acre. But if he is producing at some distance-say, 5 miles - and it costs him $5 per mile to ship an acre's product, his transport costs will be $25 per acre. His profits will be equal to value minus production costs minus shipping charges: 100-50-25 = $25. This relation may be shown diagrammatically (see Figure 1). At the market, the farmer's profits are $50, and 5 miles out, $25; at intermediate distance, he will receive intermediate profits. Finally, at a distance of 10 miles from the market, his production costs plus shipping charges will just equal the value of his produce at the market. At distances greater than 10 miles, the farmer would operate at a loss.

In this model, the profits derived by the farmers are tied directly to their location. If the functions of farmer and landowner are viewed as separate, farmers will bid rents for land according to the profitability of the location The profits of the farmer will therefore be shared with the landowner through rent payments. As farmers bid against each other for the more profitable locations, until farmers' profits are

[4] *A full development of the theory is presented in my doctoral dissertation*, A MODEL OF THE URBAN LAND MARKET: LOCATIONS AND DENSITIES OF DWELLINGS AND BUSINESSES, *University of Pennsylvania, 1960.*

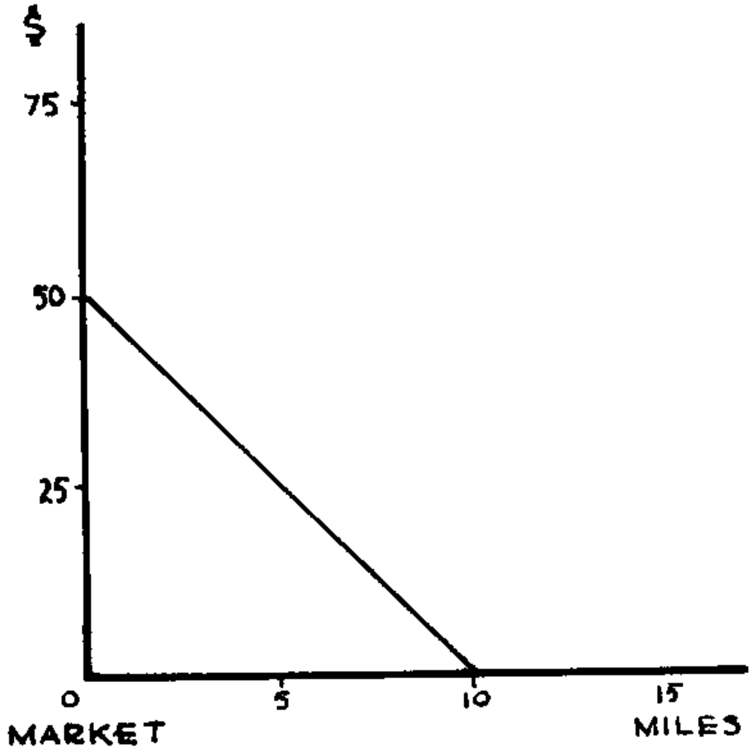

FIGURE I

everywhere the same ("normal" profits), what we have called profits becomes rent. Thus, the curve in Figure 1, which we derived as a farmers' profit curve, once we distinguish between the roles of the farmer and the landowner, becomes a bid rent function, representing the price or rent per acre that farmers will be willing to pay for land at the different locations.

We have shown that the slope of the rent curve will be fixed by the transport costs on the produce. The level of the curve will be set by the price of the produce at the market. Examine Figure 2. The lower curve is that of Figure 1, where the price of wheat is $100 at the market, and production costs are $50. If demand increases, and the price of wheat at the market rises to $125 (while production and transport costs remain constant), profits or bid rent at the market will be $75; at 5 miles, $50; $25 at 10 miles, and zero at 15 miles. Thus, each bid rent curve is a function of rent vs. distance, but there is a family of such curves, the level of any one determined by the price of the produce at the market, higher prices setting higher curves.

Consider now the production of peas. Assume that the price at the market of one acre's production of peas is $150, the costs of production

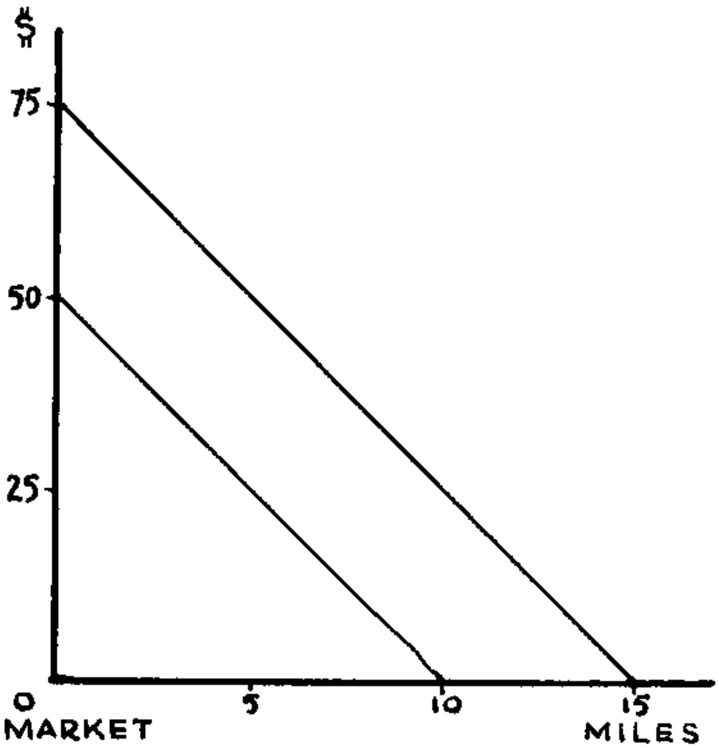

FIGURE 2

are $75, and the transport costs per mile are $10. These conditions will yield curve MN in Figure 3, where bid rent by pea farmers at the market is $75 per acre, 5 miles from the market $25, and zero at 7.5 miles. Curve RS represents bid rents by wheat farmers, at a price of $100 for wheat. If will be seen that pea farmers can bid higher rents in the range of 0 to 5 miles from the market; farther out, wheat farmers can bid higher rents. Therefore, pea farming will take place in the ring from 0 to 5 miles from the market, and wheat farming in the ring from 5 to 10 miles. Segments MT of the bid rent curve of pea farming and TS of wheat farming will be the effective rents, while segments RT and TN represent unsuccessful bids.

The price of the product is determined by the supply-demand relations at the market. If the region between zero and 5 miles produces too many peas, the price of the product will drop, and a lower bid rent curve for pea farming will come into effect, so that pea farming will be practiced to some distance less than 5 miles.

Abstracting this view of the agricultural land market, we have that:

(1) land uses determine land values, through competitive bidding among farmers;

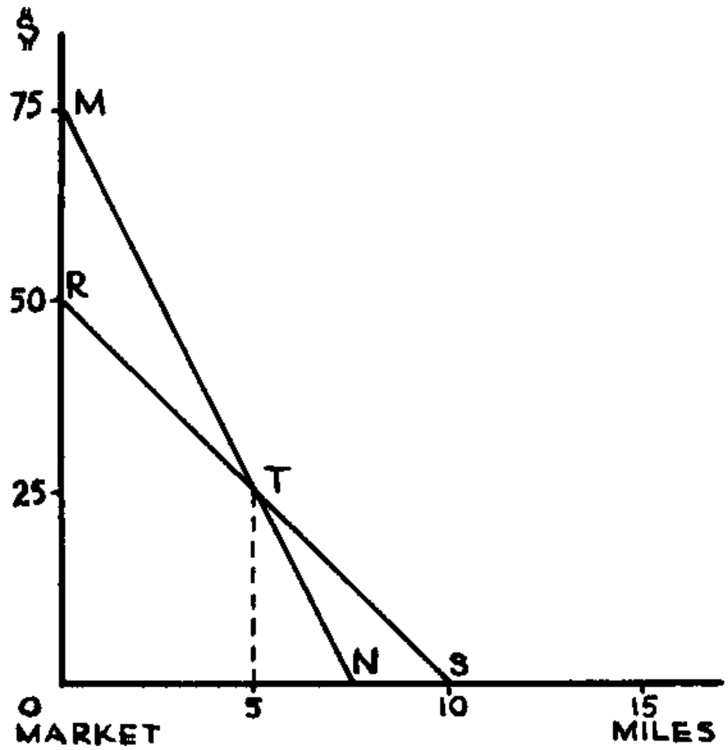

FIGURE 3

(2) land values distribute land uses, according to their ability to pay;
(3) the steeper curves capture the central locations. (This point is a simplified one for simple, well-behaved curves.

Abstracting the process now *from* agriculture, we have:

(1) for each user of land (e.g., wheat farmer) a family of bid rent functions is derived, such that the user is indifferent as to his location along any *one* of these functions (because the farmer, who is the decision-maker in this case, finds that profits are everywhere the same, i.e., normal, as long as he remains on one curve);
(2) the equilibrium rent at any location is found by comparing the bids of the various potential users and choosing the highest;
(3) equilibrium quantities of land are found by selecting the proper bid rent curve for each user (in the agricultural case, the curve which equates supply and demand for the produce).

BUSINESS

We shall now consider the urban businessman, who, we shall assume, makes his decisions so as to maximize profits. A bid rent curve for the businessman, then, will be one along which profits are everywhere the same: the decision-maker will be indifferent as to his location along such a curve.

Profit may be defined as the remainder from the volume of business after operating costs and land costs have been deducted. Since in most cases the volume of business of a firm as well as its operating costs will vary with its location, the rate of change of the bid rent curve will bear no simple relation to transport costs (as it did in agriculture). The rate of change of the total bid rent for a firm, where profits are constant by definition, will be equal to the rate of change in the volume of business minus the rate of change in operating costs. Therefore the slope of the bid rent curve, the values of which are in terms of dollars per unit of land, will be equal to the rate of change in the volume of business minus the rate of change in operating costs, divided by the area occupied by the establishment.

A different level of profits would yield a different bid rent curve. The higher the bid rent curve, the lower the profits, since land is more expensive. There will be a highest curve, where profits will be zero. At higher land rents the firm could only operate at a loss.

Thus we have, as in the case of the farmer, a family of bid rent curves, along the path of any one of which the decision-maker - in this case, the businessman - is indifferent. Whereas in the case of the farmer the level of the curve is determined by the price of the produce, while profits are in all cases "normal", i.e., the same, in the case of the urban firm, the level of the curve is determined by the level of the profits, and the price of its products may be regarded for our purposes as constant.

RESIDENTIAL

The household differs from the farmer and the urban firm in that satisfaction rather than profits is the relevant criterion of optional location. A consumer, given his income and his pattern of tastes, will seek to balance the costs and bother of commuting against the advantages of cheaper land with increasing distance from the center of the city and the satisfaction of more space for living. When the individual consumer faces a given pattern of land costs, his equilibrium location and the size of his site will be in terms of the marginal changes of these variables.

The bid rent curves of the individual will be such that, for any given curve, the individual will be equally satisfied at every location at the price set by the curve. Along any bid rent curve, the price the individual will bid for land will decrease with distance from the center at a rate just sufficient to produce an income effect which will balance to his satisfaction the increased costs of commuting and the bother of a long trip. This slope may be expressed quite precisely in mathematical terms, but it is a complex expression, the exact interpretation of which is beyond the scope of this paper.

Just as different prices of the produce set different levels for the bid rent curves of the farmer, and different levels of profit for the urban firm, different levels of satisfaction correspond to the various levels of the family of bid rent curves of the individual household. The higher curves obviously yield less satisfaction because a higher price is implied, so that, at any given location, the individual will be able to afford less land and other goods.

INDIVIDUAL EQUILIBRIUM

It is obvious that families of bid rent curves are in many respects similar to indifference curve mappings. However, they differ in some important ways. Indifference curves map a path of indifference (equal satisfaction) between combinations of quantities of two goods. Bid rent functions map an indifference path between the price of one good (land) and quantities of another and strange type of good, distance from the center of the city. Whereas indifference curves refer only to tastes and not to budget, in the case of households, bid rent functions are derived both from budget and taste considerations. In the case of the urban firm, they might be termed isoprofit curves. A more superficial difference is that, whereas the higher indifference curves are the preferred ones, it is the lower bid rent curves that yield greater profits or satisfaction. However, bid rent curves may be used in a manner analogous to that of indifference curves to find the equilibrium location and land price for the resident or the urban firm.

Assume you have been given a bid rent mapping of a land use, whether business or residential (curves $brc_{1,2,3}$, etc., in Figure 4). Superimpose on the same diagram the actual structure of land prices in the city (curve SS). The decision-maker will wish to reach the lowest possible bid rent curve. Therefore, he will choose that point at which the curve of actual prices (SS) will be tangent to the lowest of the bid rent curves with which it comes in contact (brc_2). At this point will be the equilibrium location (L) and the equilibrium land rent (R) for this user of land. If he is a businessman, he will have maximized profits; if he is a resident, he will have maximized satisfaction.

Note that to the left of this point of equilibrium (toward the center of the city) the curve of actual prices is steeper than the bid rent curve; to the right of this point (away from the center) it is less steep. This is another aspect of the rule we noted in the agricultural model: the land uses with steeper bid rent curves capture the central locations.

MARKET EQUILIBRIUM

We now have, conceptually, families of bid rent curves for all three types of land uses. We also know that the steeper curves will occupy the more central locations. Therefore, if the curves of the various users are ranked by steepness, they will also be ranked in terms of their accessibility from the center of the city in the final solution.

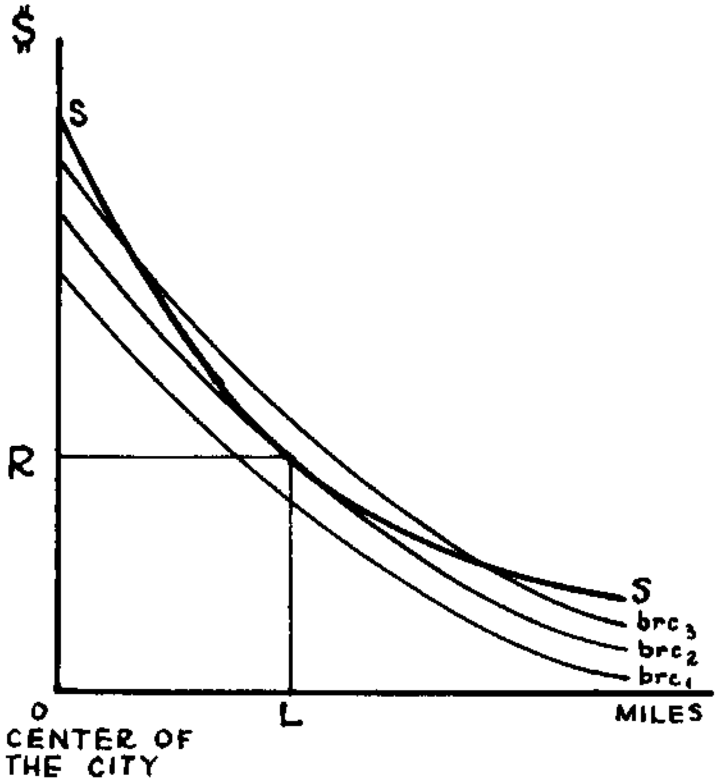

FIGURE 4

Thus, if the curves of the business firm are steeper than those of residences, and the residential curves steeper than the agricultural, there will be business at the center of the city, surrounded by residences, and these will be surrounded by agriculture.

This reasoning applies as well within land use groupings. For instance, it can be shown that, given two individuals of similar tastes, both of whom prefer living at low densities, if their incomes differ, the bid rent curves of the wealthier will be flatter than those of the man of lower income. Therefore, the poor will tend to central locations on expensive land and the rich to cheaper land on the periphery. The reason for this is not that the poor have greater purchasing power, but rather that they have steeper bid rent curves. This stems from the fact that, at any given location, the poor can buy less land than the rich, and since only a small quantity of land is involved, changes in its price are not as important for the poor as the costs and inconvenience of commuting. The rich, on the other hand, buy greater quantities of land, and are consequently affected by changes in its price to a greater degree. In other words, because of variations in density among different levels of income, accessibility behaves as an inferior good.

Thus far, through ranking the bid rent curves by steepness, we have found the relative rankings of prices and locations, but not the actual prices, locations, or densities. It will be remembered that in the agricultural case equilibrium levels were brought about by changes in the price of the products, until the amount of land devoted to each crop was in agreement with the demand for that crop.

For urban land this process is more complex. The determination of densities (or their inverse, lot size) and locations must be found simultaneously with the resulting price structure. Very briefly, the method consists of assuming a price of land at the center of the city, and determining the prices at all other locations by the competitive bidding of the potential users of land in relation to this price. The highest bid captures each location, and each bid is related to a most preferred alternative through the use of bid rent curves. This most preferred alternative is the marginal combination of price and location for that particular land use. The quantities of land occupied by the land users are determined by these prices. The locations are determined by assigning to each successive user of land the location available nearest the center of the city after the assignment of land quantities to the higher and more central bidders.

Since initially the price at the center of the city was assumed, the resulting set of prices, locations, and densities may be in error. A series of iterations will yield the correct solution. In some cases, the solution may be found by a set of simultaneous equations rather than by the chain of steps which has just been outlined.

The model presented in this paper corresponds to the simplest case: a single-center city, on a featureless plain, with transportation in all directions. However, the reasoning can be extended to cities with several centers (shopping, office, manufacturing, etc.), with structured road patterns, and other realistic complications. The theory can also be made to shed light on the effects of economic development, changes in income structure, zoning regulations, taxation policies, and other. At this stage, the model is purely theoretical; however, it is hoped that it may provide a logical structure for econometric models which may be useful for prediction.